Mathematical Methods

Programme: M. Sc. (Mathematics) Year : II Semester : III
Course: Core Credits : 4 Hours : 40

Course Context and Overview):

This course emphasizes concepts and techniques for solving integral equations from an applied mathematics perspective. To impart analytical ability in solving variational problems. The purpose is also to present an introduction to the theory of integral transforms, with emphasis on the Fourier and Laplace transforms. The mathematical methods covered by this course are the important analytic methods that are useful for modelling the real world. This course emphasizes concepts and techniques for solving integral equations, solving variational problems in calculus of variation and introduction to the theory of integral transforms, with emphasis on the Fourier and Laplace transforms from an applied mathematics perspective.

Prerequisites Courses: Analysis and Differential Equations.

Course Outcomes (COs):

On completion of this course, the students will have the ability to:
CO1: To introduce the methods and concepts to solve integral equations
CO2: Be thorough with variational problems
CO3: To introduce to the theory and application of integral transforms, with emphasis on the Fourier and Laplace transforms.

Course Topics:

Contents		Lecture Hours
UNIT – 1:		4
Definition and classification of linear integral equations. Conversion of initial and boundary value problems into integral equations. Conversion of integral equations into differential equations.		
UNIT –2:		10
Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.		
UNIT-3:		4
Basic concepts of the calculus of variations such as functional, extremum, variations, function spaces.		
UNIT-4		10
Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations. Invariance of Euler`s equations. Variational problem in parametric form.		
UNIT-5		12

Fourier Series and Fourier Transforms: Orthogonal set of functions, Fourier series, Fourier sine and cosine series, Half range expansions, Fourier integral Theorem, Fourier Transform, Fourier Cosine Transform, Fourier Sine Transform, Transforms of Derivatives, Fourier transforms of simple Functions, Fourier transforms of Rational Functions, Cosine and Sine Transforms, Inversion Theorem.	6	
Laplace Transform: Definition, Transform of some elementary functions, rules of manipulation of Laplace Transform, Transform of Derivatives, relation involving Integrals, the error function, Transform of Bessel functions, Periodic functions, convolution of two functions, Inverse Laplace Transform of simple function, Tauberian Theorems, Solution of Differential Equations	4	
Hankel Transform: Elementary properties, Inversion theorem, transform of derivatives of functions, transform of elementary functions.	2	

Textbook & References:

Text Book:

- Jerry, Abdul J., Introduction to Integral Equations with applications, Clarkson University Wiley Publishers, 2nd Revised edition edition (11 October 1999)
- Elsgolc, L.E.: Calculus of Variations, Dover Publications Inc. (15 January 2007)
- Loknath Debnath, Integral Transforms and their applications, Chapman and Hall/CRC; 2 edition , 2006.
- Donald A. Mc Quarrie: Mathematical Methods for Scientists & Engineers, University Science Books, Edition: 2008.

Reference books:

- Cordumeanu, C. : Integral Equations and Applications, Cambridge University Press, 1991.
- Curant, R. and D. Hilbert: Methods of Mathematical Physics, Vol I. Interscience Press, 1953.
- Ian N. Sneddon , The use of Integral Transforms ,McGraw Hill; Second Printing edition ,1972.