MTH 4052: Ordinary Differential Equations

Programme: M.Sc (Mathematics) Year: Ist Year Semester : Even
Course : core Credits : 4 Hours : 40

Course Context and Overview (100 words): This is a traditional course of ordinary differential equations popularly known as ODE for PG students. The main objective of this course is to introduce the students with the broad range and development in the area of ODE. In this course we will be basically covering first order differential equations, higher order differential equations and systems, asymptotic behavior of solutions, power series solutions. In addition we also have plan to introduce certain notions and especially some topics like: phase plane analysis, classification of critical points, Asymptotic behavior and stability of linear systems, linearized stability and Lyapunov methods for the two dimensional autonomous systems and phase space. Thus this course is going to be very useful for the MSc students.

Prerequisites Courses: Basic Calculus, Linear Algebra

Course outcomes(COs):

On completion of this course, the students will have the ability to:
CO1 Determine whether or not a unique solution to a linear nth-order initial-value problem exists
C02 Use power series and Frobenius series techniques to solve linear differential equations in the neighbourhood of ordinary points and regular singular points respectively
C03 solve systems of homogeneous first-order linear differential equations using matrix methods
C04 Apply various principles from linear algebra to assemble solutions.
C05 find trajectories associated with, determine critical points of, and perform phase plane analysis for simple autonomous linear and non-linear systems of equations

Course Topics:

Topics		**Lecture Hours**
UNIT - I **1. Topic** Existence and Uniqueness of Solutions		
1.1 Review of solution methods for first order, Existence and uniqueness of initial value problems: Picard′s and Peano′s Theorems,		7
1.2 Gronwall′s inequality, Picard's theorem for systems,		

1.3 Continuation of solutions and maximal interval of existence, Continuous dependence		
UNIT - II **2. Topic** Linear Differential Equations of Higher Order		7
2.1 Introduction, General solution of second order and higher equations		
2.2 Higher order linear equations and linear Systems: fundamental solutions, Wronskian, variation of constants,		
2.3 Exponential matrix and asymptotic behaviour of solutions, Power series methods with properties of Legendre polynomials and Bessel functions.		
UNIT – III **3. Topic** Systems of linear differential equations, Oscillations and Boundary value Problems		15
3.1 Introduction, Systems of First Order Equations		
3.2 Fundamental Matrix, Non-homogeneous linear Systems, Linear Systems with Constant Coefficients		
3.3 Phase Portraits-Introduction, Phase Portraits in R 2 (continued)		
3.4 Sturm's Comparison Theorem, Elementary Linear Oscillations		
3.5 Sturm-Liouville Problem, Green's Functions		
UNIT – IV **4. Topic** Asymptotic behavior and Stability Theory		11
4.1 . Introduction, Linear Systems with Constant Coefficients, Linear Systems with Variable Coefficients		
4.2 Second Order Linear Differential Equations, Stability of Quasi-linear Systems, Stability of Autonomous Systems		
4.3 Stability of Non-Autonomous, A Particular Lyapunov Function and Lyapunov methods		

Textbook references (IEEE format):
Text Book:

- **G.F. Simmons,** Differential Equations with Applications and Historical Notes. New York: McGraw-Hill, 1991.
- **L. Perko,** Differential Equations and Dynamical Systems, Texts in Applied Mathematics, Vol. 7, 2nd ed., Springer Verlag, New York, 1998.
- **Fred Brauer and J.A. Nohel**, The Qualitative Theory of ordinary Diff. equations

Reference books: V Raghavendra, V Lakshmikantam, S Deo, Text book of ordinary differential equations, Tata McGraw-Hill Education, 2008

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Methods:

Item	Weightage
Quizzes/Assignments	25
Midterm	25
Final Examination	50

Prepared By: Course Instructor name : Dr. Vikas Gupta