Mathematical Physics-I

Programme: M.Sc. (Physics)	Year: 1st	Semester : 1st
Course : Core Course	Credits : 4	Hours (LTP): 40+12+0

Course Context and Overview:
Importance of mathematics in the formulation of physics is well known. This course introduces the students to some of the basic and most frequently used mathematical tools in physics. These tools find applications in almost all areas of physics from classical mechanics to electrodynamics to quantum mechanics.

Prerequisite Courses:
Basic knowledge of calculus, coordinate geometry, complex numbers etc.

Course outcomes(COs):

On completion of this course
CO1: Students will acquire an understanding of the concept of linear vector spaces and they should be in a position to use the concept and techniques in various areas of physics like quantum mechanics.
CO2: Students will have an understanding of generalized (curvilinear) coordinate systems and tensors and they should be able to use the tensorial concepts to solve problems in various areas of physics like electrodynamics. This will also be a first step towards the more advanced topics like General Relativity.
CO3: Students will be able to use Fourier series as a basic tool of analysis in almost all branches of physics.
CO4: Students would have an appreciation for the most commonly occurring ordinary differential equations in physics and the methods for solving them.
CO5: Students would be able to understand how various special functions arise in different physical problems and use their knowledge to analyze these problems using special functions.

Course Topics:

Topics	Lectures+Tutorials
Linear vector spaces: Definition, bra-ket notation, scalar product, orthogonal vectors, dual vectors, Cauchy-Schwartz inequality, real and complex vector spaces, metric spaces and norm/length of a vector, triangle inequality, linear operators and their algebra, left and right inverse of a linear operator, adjoint of an operator, Hermitian or self-adjoint operators, unitary operators, projection operator, linear independence of vectors, basis and span, components of a vector,	13+3

eigenvalues and eigenvectors, Kronecker delta, Gram-Schmidt orthogonalization theorem, N-dimensional vector space, Representation of vectors and linear operators – matrices, matrix algebra, determinant of a matrix, inverse of a matrix, change of basis in N-dimensional vector space, similarity transformation, orthogonal bases, representation of adjoint operators, notion of transpose, representation of Hermitian and unitary operators, Application to real vector space: Cartesian coordinates, Orthogonal matrices, symmetric and anti-symmetric matrices, trace of a matrix, eigenvalues and eigenvectors of matrices, spectrum and spectral radius, linear independence of eigenvectors, eigenvalues and eigenvectors of similar matrices, matrix diagonalization, simultaneous diagonalization of Hermitian matrices, Cayley-Hamilton theorem.	
Introduction to Tensor analysis: Scalars, vectors, Concept of tensors, covariant and contravariant (components of) tensors, mixed tensors, rank of a tensor, tensors in a real vector space, Cartesian and non-Cartesian tensors, algebra of tensors, symmetric and anti-symmetric tensors, changing the rank of tensors – direct product, contraction, pseudo tensors, metric tensor – raising and lowering indices of the components of tensors.	4+1
Fourier series: Periodic functions, definition of Fourier series, Dirichlet conditions, sine and cosine as eigenfunctions of a linear, self-adjoint (differential) operator, Complete orthonormal set (only definition), Euler formula for Fourier coefficients, Fourier series expansion for periodic functions with arbitrary period, Complex form of Fourier series, Even and odd functions and symmetry properties, Half range expansion, integration and differentiation of Fourier series, Gibb's phenomena, Expression of Fourier coefficients as consequence of minimization of mean square error between the function and its representation as a trigonometric series, completeness of sine and cosine functions, convergence of Fourier series.	4+2
Ordinary differential equations: Classification of differential equations, order of a differential equation, degree of a differential equation, homogeneous and inhomogeneous differential equations, notion of solution of a differential equation, initial/boundary value problem, general solution and particular solution, singular solution, linear homogeneous and inhomogeneous ordinary differential equations and principle of superposition. **First order ordinary differential equations:** separable equations, exact differential equations, non-exact equations and integrating factor, Bernoulli equation, Existence and uniqueness of solution of first order initial value problem.	3+1
Second order ordinary differential equations: homogeneous equations with constant coefficient, Euler-Cauchy equation (a homogeneous equation with variable coefficients), existence and uniqueness of solution of homogeneous initial value problem, solution of non-homogeneous equation using method of undetermined coefficient and solution by variation of parameters, Power series and Frobenius methods for solving ordinary differential equations, notion of regular or nonessential singular point and	3+1

essential singular point, Fuchs theorem.	
Sturm-Liouville theory, Green's function	3+1
Special functions: Legendre, Bessel, Laguerre and Hermite functions. Differential equation approach and generating function approach, recurrence relation, orthogonality, related functions Neumann functions or Bessel functions of the second kind, associated Legendre functions etc.	10+3

Textbooks:

1. G.B. Arfken and H.J. Weber, Mathematical Methods for Physicists (Elsevier; 7th Ed. 2012).
2. E. Kreyszig, Advanced Engineering Mathematics (Wiley; 10th edition, 2015).

Additional References:

1. P. Dennery and A. Krzywicki, Mathematics for Physicists, (Dover Publications Inc.; New edition, 1996). (Especially for Linear spaces and tensor analysis).
2. J. Mathews and R.L. Walker, Mathematical Methods of Physics, (Pearson Addison-Wesley; 2nd edition, 1971).

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

1. Video lectures on Selected topics in Mathematical Physics by Prof. V. Balakrishnan (IIT Madras) available at http://nptel.ac.in/courses/115106086/

Evaluation Method:

Item	Weightage
Quizzes and/or assignments	45
Midterm	20
Final Examination	35

Prepared By: Physics Department (BoS recommended)
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