LNMIIT, Jaipur Department of Mathematics	LNMIIT The LNM Institute of Information Technology

Programme: Ph.D. (Mathematics)	**Course Title:** **Robust Computational Techniques for Singularly Perturbed Problems**			**Course Code:** MTH-6XXX
Type of Course: Program Core	**Prerequisites:** A basic knowledge of numerical analysis and differential equations is helpful.			**Total Contact Hours:** 45L+15T
Year/Semester:	**Lecture (L) Hrs/Week:** 3	**Tutorial (T) Hrs/Week:** 1	**Practical(P) Hrs/Week:** 0	**Credits:** 4

Learning Objective:

This is an introductory course on basic theoretical and computational aspects of the singularly perturbed problems (SPPs). We will start with the introduction and motivation for the study of SPPs and their applications, layer behavior of SPPs, layer adapted meshes. In particular, we will concentrate ourselves for the study of finite difference method (FDM) and finite element method (FEM) on layer adapted meshes for reaction-convection-diffusion problems. This course serve as an introductory course for the theory and numerics of SPPs and student can lead to pursue research in this area with the gain knowledge in this course.

Course outcomes (COs):

On completion of this course, the students will have the ability to:		**Bloom's Level**
CO-1	Understanding with the basics of the SPPs and layer adapted meshes	
CO-2	Solve the resulting problem classes efficiently by computational methods based on a sound mathematical analysis	
CO-3	Pursue further research in the allied areas	

Course Topics	Lecture Hours	
UNIT – I **Topics: Introduction to SPPs and Layer-Adapted Meshes**		**8**
1.1) Singular and regular perturbation problems, Motivation for the study of SPPs, Simple examples of SPPs, Concept of boundary/interior layers, Uniform Convergence of numerical methods,	3	
1.2) Layer-Adapted Meshes – Mesh generating function, Mesh equidistribution, Bakhvalov, Shishkin and Shishkin type meshes for 1D convection-diffusion problems with boundary layers, Interior layers, 2D reaction-diffusion problem with boundary layers, 2D convection-diffusion problems with regular boundary, parabolic boundary and characteristic interior layers	5	
UNIT – II **Topics: Analytical Behaviour and numerical solution for 1D Singularly Perturbed Problems**		22
2.1) Preliminaries – Stability of differential operators, Green's functions, Comparison principle for discrete operator, M-Matrix criterion, Convection-Diffusion Problems with Regular Layers -- Stability and Green's function estimates, Derivative bounds and Solution decomposition, Convection-Diffusion Problems with Turning-Point Layers	**7**	
2.2) Finite Difference Methods – A simple upwind difference scheme, Second-order difference schemes, Problems with turning point layers,	7	
2.3) Finite Element Methods – The interpolation error, Linear Galerkin FEM, Stabilised FEM, Artificial viscosity stabilisation, Streamline-diffusion stabilisation	8	
UNIT – III **Topics: Analytical Behaviour and numerical solution for 2D Singularly Perturbed**		**15**

Problems		
3.1) Preliminaries – Stability of the general elliptic second-order differential operator, Comparison principle, Compatibility conditions at corners for regularity of solutions, Convection-diffusion problems – Regular layers, Characteristic layers	3	
3.2) FDM for Convection-diffusion problems – Upwind difference schemes, Stability, Pointwise error bounds, Error expansion	5	
3.3) FEM for Convection-diffusion problems – The interpolation error, Galerkin FEM, Artificial Viscosity stabilisation, Streamline-diffusion FEM, Characteristic layers	7	

Textbook References:
Text Book:

1. Layer-Adapted Meshes for Reaction-Convection-Diffusion Problems by Torsten Linss, Lecture Notes in Mathematics, Springer (2010)
2. Robust Numerical Methods for Singularly Perturbed Differential Equations by HG Roos, Martin Stynes and Lutz Tobiska, Springer series in Computational Mathematics, Springer (2008).

Reference Book:

3. Fitted Numerical Methods for Singular Perturbation Problems: Error Estimates in the Maximum Norm for Linear Problems in One and Two Dimensions by JJH Miller, E O'Riordan and GI Shishkin, World Scientific (2012)

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Method	
Items	**Weightage (%)**
Presentation	40
Midterm	20
End-Term	40

CO and PO Correlation Matrix

CO	PO1	PO2	PO3	PO4	PSO1	PSO2	PSO3
CO1							
CO2							
CO3							
CO4							

S- Strong; M-Medium; L-Low

Last Updated On: Jan 2023

Updated By: Vikas Gupta and Sudipto Chowdhury
Approved By: