Programme: Ph. D (Mathematics)	Course Title: Sobolev Spaces, Weak Formulation and Finite Element Methods			Course Code:
Type of Course: Elective	Prerequisites: Numerical Analysis, Functional Analysis.			Total Contact Hours: 60
Year/Semester:	Lecture Hrs/Week:	Practical Hrs/Week:	Credits: 4	

Learning Objective:

In this course we introduce the modern theory of linear elliptic partial differential equations (pdes) and its applications to develop the basic mathematical theory of finite element methods. We introduce function spaces related to the solutions of pdes, known as Sobolev spaces and subsequently variational/ weak formulations of pdes. We study various properties of distributional derivative, Sobolev spaces. Various conditions on the existence and uniqueness of the weak solutions of second and fourth order pdes are discussed. Subsequently the discretization concept is studied using finite element method. Construction of various finite element spaces based on triangular elements and piecewise polynomial approximation of Sobolev functions are studied. This approximation theory is used to approximate the solutions of various second and fourth order problems.

Prerequisites of the course:

- Numerical Analysis
- Functional Analysis

Course outcomes (COs):

On completion of this course, the students will have the ability to:		Bloom's Level
CO-1	Understand the basics of existence, uniqueness and regularity of the solutions of various second and fourth order elliptic pdes.	3, 4
CO-2	Understand basic theory, importance, efficiency of finite element methods	4, 5
CO-3	Construct finite element spaces to solve various pdes numerically.	4, 5, 6
CO-4	Derive error estimates for the solutions in L^{2} and energy norm.	4, 5, 6

Course Topics	Lecture Hours	
UNIT – I		13
Idea of distributional derivative, some operations with distributions. Sobolev spaces, basic properties, Poinc`are inequality, Trace theorems, Sobolev embedding, Green's theorem.		
UNIT – II		12
Abstract variational formulation of elliptic boundary value problem. Lax Milgram Lemma, Inf Sup condition, Galerkin formulation and Cea's lemma.		

Construction of various Lagrange and Hermite finite element spaces for second and fourth order boundary value problems.		
UNIT – III		**17**
3.1 General considerations on convergence, Interpolation theory in Sobolev spaces and polynomial approximation. 3.2 Conforming finite element for second order problems. Error estimate in energy norm. Aubin Nitsche duality argument and error estimate in L^{2} norm.		
UNIT-IV		**18**
Other finite element methods for second order elliptic problems. The effect of numerical integration. First Strang's lemma. Sufficient condition for uniform ellipticity. Error analysis		
A non conforming method, non conforming methods for second-order boundary value problems. Abstract error estimate, Second Strang's lemma. Error analysis.		

Textbook References:

Text Book:

Text Books

1. Topics in Functional Analysis and Applications. S. Kesavan, New Age publications, 2003.

2. P.G. Ciarlet, The Finite Element Method for Elliptic Problems, SIAM: Society for Industrial and Applied Mathematics; 2nd edition, April 2002.

3. S. C. Brenner and L. R. Scott, The mathematical theory of finite element methods, 3rd ed., Springer, 2008.

Reference books:

Claes Johnson, Numerical Solution of Partial Differential Equations by the Finite Element
Method, Dover Publications, 2009.

D. Braess, Finite elements: Theory, fast solvers, and applications in solid mechanics, Cambridge 2007.

Additional Resources:
NPTEL, MIT Video Lectures.

Evaluation Method	
Item	**Weightage (%)**
Presentation	40
Midterm	20
End-Term	40

CO and PO Correlation Matrix

CO	PO 1		P O2 P O3 P O4		PSO 1	PSO 2	PSO 3
CO1	3	2	2	3	3	3	3
CO2	3	1	3	3	3	3	3
CO3	3	3	3	3	3	3	3
CO4	3	2	2	3	3	3	3

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Approved By: