Programme: Ph.D. (Math.)	Course Title: Advanced Finite Element Method			Course Code: MTH-
Type of Course: Elective	Prerequisites: Functional Analysis, Numerical Analysis, Partial Differential Equations , Sobolev Space			Total Contact Hours: 45+15
Year/Semester: NA	Lecture Hrs/Week: 3	Tutorial Hrs/Week: 1	Practical Hrs/Week:	Credits: 4

Learning Objective:

The objective of the course is to understand the deep theoretical analysis of finite element methods. Choosing an approximation space and appropriate finite element formulation are crucial in applying finite element methods. Different finite element may be used depending on the nature of the PDEs, and the underlying domain. In this course we study how to develop mathematical theory of finite element method, construct finite element function spaces based on triangular or rectangular finite elements and piecewise polynomials approximation. Further, convergence analysis with respect to various norms will be studied in details for various finite element methods to solve elliptic and parabolic PDEs.

Course outcomes (COs):

On completion of this course, the students will have the ability to:		Bloom's Level
CO-1	**construct** various finite elements and finite element function spaces	3
CO-2	**develop** weak and discretize solutions of PDEs and show the existence and uniqueness of those solutions.	4
CO-3	**analyze** the convergence analysis to identify the robustness and possible improvement of the finite element methods.	4
CO-4	**evaluate** finite element methods to various elliptic and parabolic PDEs arising from various physical systems.	5

Course Topics	Lecture Hours	
UNIT – I	**15**	**15**
The three basic aspects of the finite element method, Examples of simplicial and rectangular finite elements and their associated finite element spaces, finite elements with derivatives as degrees of freedom and their associated finite element spaces, finite elements for fourth-order problems and their associated finite element spaces, Finite elements as triplet and their associated interpolation operators, Affine families of finite elements, General properties of finite element spaces, General considerations on the convergence of finite element methods and Cea's lemma.	15	
UNIT – II	**15**	**15**
Finite Element Methods for Second-Order Problems: The Basic Error Estimates: Estimate of the seminorms for polynomial-preserving operators, Estimate of the interpolation errors for an affine family of finite elements, Interpolation and approximation properties of finite element spaces, Error estimates with respect to W^{m,p}norms, maximum norm and negative norm.	**15**	
UNIT – III	**15**	**15**
Nonconforming Finite Element Methods for Second-Order Problems: Nonconforming methods, Abstract error estimate and the second Strang lemma, Example of a nonconforming finite element, Consistency error estimate, Estimate of the error.	15	
UNIT – IV	**15**	**15**
Parabolic initial and boundary value problems: semidiscrete and completely discrete schemes, Existence and uniqueness of the discrete solutions, Error estimates, superconvergence, maximum norm estimates, semigroup theory, solution operators.	15	

Textbook References:
Text Book:

1. P.G. Ciarlet, The Finite Element Method for Elliptic Problems, SIAM: Society for Industrial and Applied Mathematics; 2nd edition, April 2002.
2. Vidar Thomee, Galerkin Finite Element Methods for Parabolic Problems; Springer Berlin Heidelberg, 2006.9.

Reference books:

1. S. C. Brenner and L. R. Scott, The mathematical theory of finite element methods, 3rd ed., Springer, 2008.

Additional Resources:

Evaluation Method	
Item	**Weightage (%)**
Presentation	40
Midterm	25
End-Term	35

CO and PO Correlation Matrix

CO	PO1	PO2	PO3	PO4	PSO1	PSO2	PSO3
CO1							
CO2							
CO3							
CO4							

Last Updated On: 2022

Updated By:

Approved By: