MTH 5041: Numerical Methods for Partial Differential Equations

Programme: M.Sc (Mathematics) Year: IInd Year Semester: 3rd
Course : Elective Credits : 4 Hours : 40

Course Context and Overview (100 words): The physical world is described by a small set of laws. One of the most useful mathematical statements of these physical laws is in the form of partial differential equations (PDE's) describing local changes in physical parameters. Though it is usually straightforward to write down the particular PDE's that correspond to a given problem, finding a solution is much more difficult. Therefore, objective of this course is to introduce the basics principles for constructing numerical schemes for the solution of PDEs, in particular finite difference schemes. This course is designed for PG students to learn the basic theories and algorithms of finite difference methods for parabolic, hyperbolic and elliptic PDE's.

Prerequisites Courses: Basic Calculus, Elementary Linear Algebra and Numerical Analysis, Computer Programming (Matlab Recommended)

Course outcomes (COs):

On completion of this course, the students will have the ability to:
CO1 learn to make a connection between the mathematical equations or properties and the corresponding physical meanings.
C02 learn the principles for designing numerical schemes for PDEs, in particular, finite difference schemes.
C03 to analyze the consistency, stability and convergence of a numerical scheme.
C04 know, for each type of PDEs (hyperbolic, parabolic and elliptic), what kind of numerical methods are best suited and the reasons behind these choices.
C05 use a programming language or Matlab to implement and test the numerical schemes.

Course Topics:

Topics	**Lecture Hours**	
UNIT - I **1. Topic** Introduction and Parabolic PDE		
1.1 Basic linear algebra – vector and matrix norms and related theorems, Some physics behind the PDEs, Introduction to finite differences,		12
1.2 Parabolic equations in 1-D: Explicit and implicit finite difference schemes,		

1.3 Truncation error and consistency, Stability analysis (matrix method, maximum principle, Fourier analysis, energy method),		
UNIT - II **2. Topic** 2D Parabolic Problems		
2.1 Maximum principle and convergence, Lax equivalence theorem, general boundary conditions, split operator methods,		08
2.2 multilevel difference schemes, nonlinear problems,		
2.3 Parabolic equations in 2D: explicit and implicit methods, ADI methods,		
UNIT – III **3. Topic** Hyperbolic PDEs		
3.1 Hyperbolic Equations: Vector systems and characteristics, Shock formation, The entropy condition, domain of dependence,		
3.2 The CFL theorem, Dissipation and dispersion, Upwind scheme, Lax-Wendroff scheme, Lax-Friedrich scheme, Leap-Frog scheme,		10
3.3 conservative schemes, Non-linear system,		
3.4 Artificial viscosity, Conservation properties, MacCormack's scheme, Richmyer method,		
3.5 Conservation laws in two space dimensions.		
UNIT – IV **4. Topic** Elliptic PDE		
4.1 .Introduction, Solvability, Maximum principle, Dirichlet, Neumann and mixed problems,		10
4.2 Basic iterative schemes, Direct factorization methods and successive over-relaxation (S.O.R.), ADI and conjugate gradient methods.		

Textbook references (IEEE format):
Text Book:

K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations,* Cambridge, 2nd Edition.

Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 1995.

LeVeque, Randall J. *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems*. SIAM 2007.

Finite Volume Methods for Hyperbolic Problems. Cambridge texts in applied mathematics. Cambridge, UK: Cambridge University Press, 2002.

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Methods:

Item	Weightage
Quizzes/Assignments	10
Midterm	30
Final Examination	60

Prepared By: Course Instructor name : Dr. Vikas Gupta
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