MTH7031 : Partial Differential Equations

Programme: M.Sc.	Year: 2019	Semester: Third
Course : Core	Credits : 4	Hours : 40

Course Context and Overview (100 words):
The main objective of the course is to introduce elements of partial differential equations for the master students. First order linear and quasi-linear PDEs, The Cauchy problem, Second order PDEs, Classification of PDEs, Characteristics, Well-posed problems, Fourier Series, Solutions of hyperbolic, parabolic and elliptic equations, Dirichlet and Neumann problems, Maximum principles, Fourier Transform methods for PDEs, The method of Green's functions for Laplace, Heat and wave equations.

Prerequisites Courses: ODE

Course outcomes (COs):

On completion of this course, the students will have the ability to:
CO1 Identify & classify PDEs of different types, solve first order PDEs and give their geometric interpretations.
C02 Compute canonical form of a second order PDE, De-Alembert solutions for PDEs.
C03 Solve Heat, Wave & Laplace equations using Method of separation of variables.
C04 Understand maximum and minimum principle, Green's identity and fundamental solution of PDEs
C05 Solve PDEs through Fourier Transform Methods and understand the Green's Functions for the PDEs.

Course Topics:

Topics	**Lecture Hours**	
UNIT - I **1. Topic**		
Mathematical Preliminaries: A Review of Multivariable Calculus, Essential Ordinary Differential Equations, Integral Curves and Surfaces of Vector Fields, Solving Equations of the form: dx/P=dy/Q=dz/R.	3	11

1.2 **First-Order Partial Differential Equations(PDEs)**– Formation and classification of first-order PDEs, Linear and Quasi-linear first-order PDEs, Cauchy's problem for first order PDEs, The Cauchy Kowalevski Theorem, Integral surfaces passing through a given curve, Nonlinear first-order PDEs, The method of characteristics, Compatible systems, Charpit's method, Jacobi's method for nonlinear PDEs.	8	
UNIT - II **2. Topic**		10
Second-Order PDEs - Classification, Canonical forms, Well-posed problems, Superposition principle. Fourier Series.	5	
The Heat Equation - Derivation of the heat equation, The maximum and minimum principles, Uniqueness, Continuous dependence, Method of separation of variables, Time-independent boundary conditions, Time-dependent boundary conditions, Duhamel's principle.	5	
UNIT - III **3. Topic**		11
3.1 **The Wave Equation** - Derivation of the wave equation, The infinite string problem, The D'Alembert solution of the wave equation, The semi-infinite string problem, The finite vibrating string problem, The method of separation variables, The inhomogeneous wave equation.	5	
3.2 **Laplace's Equation** – Basic concepts, Types of boundary value problems, The maximum and minimum principle, Green's identity and fundamental solution, The Poisson integral formula, The method of separation of variables, The Dirichlet problem for the rectangle, The Dirichlet problem for Annuli and Disk, The exterior Dirichlet problem.	6	
UNIT - IV **4. Topic**		8
4.1 **The Fourier Transform Methods for PDEs** – Fourier transform, Fourier sine and cosine transform, Heat flow problem in an infinite and semi-infinite rod, Infinite string problem, Laplace equation in a half-plane.	5	
4.2 **The Method of Green's Functions** – Integral formulation, The method of Green's functions for the Laplace, Heat and Wave equations.	3	

Textbook references (IEEE format):

Text Book:

1. Ian N. Sneddon, Elements of Partial Differential Equations, Dover Publications, 2006.
2. F. John, Partial Differential Equations, 3rded., Narosa Publ. Co., New Delhi,1979.

Reference books:

1. T. Amarnath, An Elementary Course in Partial Differential Equations.
2. L.C. Evans, Partial Differrential Equations, Graduate Studies in Mathematics, Vol.19, AMS, Providence, 1998..

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.): NPTEL, MIT Video Lectures.

Evaluation Methods:

Item	Weightage
Quiz	20%
Midterm	30%
Final Examination	50%

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