MTH 5021: Functional Analysis

Programme: M.Sc (Mathematics) Year: 2nd Year Semester : Odd
Course : Core Credits : 4 Hours : 40

Course Context and Overview (100 words): This is a traditional course of Functional Analysis for PG students of Mathematics. The main objective of this course is to introduce the students with the broad range and development in the area of Functional Analysis starting from scratch. In this course we will be basically covering three main theorems: Uniform boundedness principle, closed graph theorem, open mapping theorem. In addition we also introduce elementary theory of bounded operators and spectral theorem for compact operators.

Prerequisites Courses: Analysis-I

Course outcomes(COs):

On completion of this course, the students will have the ability to:
CO1 Determine whether the given linear operator is bounded or not.
C02 Applications of Hahn-Banach extension theorem
C03 Several examples of Banach spaces
C04 Find different spectrums of a given bounded linear operator

Course Topics:

Topics		**Lecture Hours**
UNIT - I **1. Topic** Fundamentals of normed linear spaces		
1.1 Normed linear spaces, 1.2 Riesz lemma, characterization of finite dimensional spaces, 1.3 Banach spaces,		5
UNIT - II **2. Topic** Bounded linear maps on a normed linear space		5
2.1: Examples, linear map on finite dimensional spaces,		

2.2 finite dimensional spaces are isomorphic, operator norm. 2.3 Hahn-Banach theorems: Geometric and extension forms and their applications.		
UNIT – III **3. Topic** Three main theorems on Banach spaces		
3.1 Uniform boundedness principle, closed graph theorem, open mapping theorem, 3.2 projection, comparable norms. 3.3 Dual spaces and adjoint of an operator: Duals of classical spaces, weak and weak* convergence, adjoint of an operator.		10
UNIT – IV **4. Topic** Hilbert spaces		
• 4.1. Inner product spaces, orthonormal set, Gram-Schmidt ortho-normalization, Bessel's inequality, 4.2 Orthonormal basis, Separable Hilbert spaces. Projection and Riesz representation theorem: Orthonormal complements, 4.3 orthogonal projections, projection theorem, Riesz representation theorem.		8
UNIT – V **5 Topic** Bounded operators on Hilbert spaces		
• 5.1 Adjoint, normal, unitary, self adjoint operators, 5.2 compact operators, eigen values, eigen vectors.		6
UNIT – VI **6 Topic** Spectral theorem		
6.1 Spectral theorem for compact self adjoint operators, 6.2 statement of spectral theorem for bounded self adjoint operators.		6

Suggested Readings:

- **B. V. Limaye,** Linear Functional Analysis for Scientists and Engineers, Springer (2016). **[Textbook]**

- **K. Yoshida,** Functional Analysis, Springer.
- **S. Nanda and B. Choudhari,** Functional Analysis With Application, New Age International Ltd.
- **S. C. Bose,** Introduction to Functional Analysis, Macmillan India Ltd.

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Methods:

Item	Weightage
Quizzes(2)	20%
Midterm	30%
Final Examination	50%

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