MTH6032: Complex Analysis

Programme: M.Sc (Mathematics) Year: Ist Year Semester : Even
Course : core Credits : 4 Hours : 40

Course Context and Overview (100 words): This is a traditional course on Complex Analysis for PG students. When the real numbers are replaced by the complex numbers in the definition of the derivative of a function, the resulting complex differentiable functions turn out to have many remarkable properties not enjoyed by their real counterpart. These functions, usually known as analytic functions, have numerous applications in areas such as engineering, physics, differential equations to name just a few. Definitions and proofs will be stressed throughout the course. Thus this course is going to be very useful for the MSc students.

Prerequisites Courses: Analysis-I

Course outcomes(COs):

On completion of this course, the students will have the ability to:
CO1 Demonstrate understanding of the basic concepts underlying complex analyis.
C02 Prove basic results in complex analysis.
C03 Apply the methods of complex analysis to evaluate definite integrals and infinite series.
C04 Demonstrate skills in communicating mathematics orally and in writing.

Course Topics:

Topics	**Lecture Hours**	
UNIT - I **1. Topic** Complex Numbers and Analytic Functions		
1.1 The Algebra of Complex Numbers, Point Representation of Complex Numbers, Vectors and Polar Forms, The Complex Exponential, Powers and Roots, Inequalities involving complex numbers, Planar Sets, The Riemann Sphere and Stereographic Projection	3	13
1.2 Functions of a Complex Variable, Limits and Continuity, Differentiability, Necessary and sufficient conditions for differentiability (Cauchy-Riemann equations), Polar form of CR equations, Analyticity, Polynomials, Rational functions ,Harmonic Functions	7	
1.3 The Exponential, Trigonometric, and Hyperbolic Functions, The Logarithmic Function, Multivalued Functions, Branch cut and branch point, Complex exponent, Inverse Trigonometric and hyperbolic functions	3	
UNIT - II		9

2. Topic Complex Integrals		
2.1 Contours, Contour Integrals, Line intergrals, Rectifiable arcs, Line integrals as functions of arcs, Independence of path, ML Inequality,	2	
2.2 Antiderivatives, Cauchy-Goursat theorem - Deformation of contours approach, Vector analysis approach, Simply and Multiply connected domains	2	
2.3 Cauchy's Integral Formula and its consequences, Higher derivatives, Morera's Theorem, Cauchy inequality, Liouville's theorem and the Fundamental Theorem of Algebra, Maximum Modulus principle	5	
UNIT – III **3. Topic** Series representation for Analytic functions and Residue Theory		8
3.1 Sequences and Series, Taylor Series, Power Series, Mathematical Theory of Convergence, Laurent Series,	3	
3.2 Isolated singular point, Residues, Residue Theorem, Isolated singular points, The Point at Infinity, Residue at poles, Residue at infinity, Zeros of analytic functions, Behavior of a function in the neighborhood of an isolated singularity	5	
UNIT – IV **4. Topic** Application of Residues and Conformal Mapping		10
4.1 . Trigonometric Integrals over $[0, 2\pi]$, Improper Integrals of Certain Functions over $(-\infty,\infty)$, Jordan's Lemma, Improper Integrals Involving Trigonometric Functions, Indented Contours, Integrals Involving Multiple-Valued Functions, The Argument Principle and Rouché's Theorem	05	
4.2 Invariance of Laplace's Equation, Geometric Considerations, Linear transformation, The linear group, The cross ratio, Möbius Transformations, The Schwarz-Christoffel Transformation	05	

Textbook references (IEEE format):
Text Book:

- **J.B. Conway,** Functions of one complex variable, Narosa, New Delhi.
- **R.V. Churchill and J.W. Brown,** Complex Variables and Applications. Wiley

Reference books:

- **Lars V. Ahlfors,** Complex Analysis, McGraw-Hill International Edition.
- **T.W. Gamelin,** Complex Analysis, Springer International Edition, 2001.

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Methods:

Item	Weightage
Quizzes/Assignments	20
Midterm	30
Final Examination	50

Prepared By: Course Instructor name : Dr. Vikas Gupta
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