MTH 4031 An Introduction to Fractional Calculus

Programme: M.Sc (Mathematics) Year: IInd Year Semester : Odd
Course : Other Elective Credits : 4 Hours : 40

Course Context and Overview (100 words): The concept of integer order derivatives and integrals are well known. Fractional calculus is a branch of mathematical analysis that studies how to generalize the concept of integer order derivatives and integrals to fractional order or even a complex order derivatives and integrals. Fractional Calculus deals with derivative and integrals of arbitrary order. In recent years considerable interest in fractional calculus has been stimulated by the applications that finds in different areasof physics, chemistry, biology and engineering,The main purpose of this course is to making the students familiar with the elements of non-integer order calculus. This course also will help to develop students' practical skills to solve fractional differential equations (FDEs) and to interpret solutions. In this course, students will get acquaint with the applications of the fractional calculus and the fractional differential equations theory.

Prerequisites Courses: Basic Calculus and knowledge of Integral transform

Course outcomes(COs):

On completion of this course,
CO1 students will be familiar with the basics of fractional calculus.
C02 students will have ability to solve different types of non-integer order differential equations.
C03 students will have basic knowledge about applications of fractional calculus.

Course Topics:

Topics	**Lecture Hours**
UNIT - I **1. Topic** Special Functions of the Fractional Calculus	
1.1 Gamma function : Some properties of the gamma function, Limit representation of the Gamma function, Beta function 1.2 Mittag-Leffler function : Definition and relations to some other functions, The Laplace transform for Mittag-Leffler function, Derivatives of the Mittag-Leffler function, Differential equation for the Mittag-Leffler function 1.3 Wright function : Integral representation, Relation to other functions	9
UNIT - II **2. Topic** Fractional Derivatives and Integrals	13
2.1 Grunwald Letnikov. Grunwald Letnikov	

Fractional Derivatives. 2.2 Riemann-Liouville Fractional Derivatives. 2.3 Some Other Approaches. Caputo fractional derivatives, Generalized functions approach 2.4 Geometric and Physical Interpretation of Fractional Integration and Fractional Differentiation. 2.5 Sequential Fractional Derivatives. Left and Right Fractional Derivatives. 2.6 Properties of Fractional Derivatives. 2.7 Laplace Transforms of Fractional Derivatives. 2.8 Fourier Transforms of Fractional Derivatives. Mellin Transforms of Fractional Derivatives.	
UNIT – III **3. Topic: Fractional Differential Equations**	9
3.1 Existence and Uniqueness Theorems: Fractional Differential Equation of a General Form. Existence and Uniqueness Theorem as a Method of Solution. Dependence of a Solution on Initial Conditions. 3.2 The Laplace Transform Method: Standard Fractional Differential Equations. Ordinary and partial linear differential equations 3.3 Sequential Fractional Differential Equations. Fractional Green's functions 3.4 The Mellin Transform Method. Power Series Method. Babenko's Symbolic Calculus Method. Method of Orthogonal Polynomials.	
UNIT – IV **4. Topic: Numerical Solution of Fractional Differential Equations**	9
4.1 Numerical Evaluation of Fractional Derivatives. 4.2 Numerical Solution of Fractional Differential Equations	

Textbook references (IEEE format):
Text Book:

- **I. Podlubny,** Fractional Differential Equations, Academic Press, 1998
- **B. Guo, X. Pu and F. Huang,** Fractional Partial Differential Equations and Their Numerical Solutions, World Scientific, 2015

Reference books:

- H.J. Haubold and A.M. Mathai, An Introduction to Fractional Calculus, Nova Science Publishers, 2017.

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.):

Evaluation Methods:

Evaluation criteria will be shared by the concerned course instructor.

Prepared By: Course Instructor name : Dr. Vikas Gupta
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